Mathematics I

010

29th Oct.2014 **08.30am-11.30am**

REB
Rwanda Education Board

ORDINARY LEVEL NATIONAL EXAMINATIONS 2014

SUBJECT **: MATHEMATICS I**

DURATION **: 3 HOURS**

INSTRUCTIONS:

1) Do not open this paper until you are told to do so.
2) This paper has **TWO** sections **A** and **B.**
   - **SECTION A:** Attempt **ALL** questions. **(55 marks)**
   - **SECTION B:** Answer any **THREE** questions. **(45 marks)**
3) You may use mathematical instruments and calculators **where necessary.**
4) USE A **BLUE or BLACK INK PEN ONLY** TO WRITE YOUR ANSWERS AND A PENCIL TO DRAW DIAGRAMS.
5) SHOW CLEARLY ALL THE WORKING. **Marks will not be awarded for answers without all working steps.**

SECTION A: ATTEMPT ALL QUESTIONS. **(55 marks)**

1. Simplify: $0.42^2 - 0.58^2$ without using a calculator. **(3 marks)**

2. Simplify the fractions completely: $(2\frac{2}{5} \div 1\frac{6}{10}) \times 0.02$ **(3 marks)**

3. Find the inverse function of $g(x) = 3+4x$. **(2 marks)**

4. A piece of land is represented by a rectangle of $300cm^2$ on a map. Determine the actual area of land in hectares (ha). The scale is 1:15,000. **(3 marks)**

5. Solve: $\frac{2x-5}{x^2-4} = \frac{5}{x-2}$ **(5 marks)**

6. To pass a certain interview, a candidate must pass both oral test (R) and a written test (W). Of the candidates who attended the interview 70% passed R, 65% passed W and 15% passed R but not W. Four candidates failed both tests. How many candidates passed the interview? **(4 marks)**

7. Solve the following simultaneous equations: **(4 marks)**
   $2x + 3y = 5$
   $3x + 2y = 10$

8. Solve the following inequality: $2x - \frac{1}{3}(4x - 1) < \frac{3}{4} + x$. Illustrate the solution on the number line. **(4 marks)**

9. Calculate the distance between points A(-5, 4) and B(3, 2). **(4 marks)**

10. Find the magnitude of vector $\vec{z} + 2\vec{w}$, given that $\vec{z} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. **(4 marks)**

11. (a) Find the sum of the interior angles of a pentagon. **(2 marks)**

    (b) The sum of the interior angles of a polygon is 900°. How many sides has the polygon? **(2 marks)**

12. The longest side of a right angled triangle is 15cm and the other two sides are x cm and (x +3) cm respectively. Find x and calculate the area of the triangle. **(4 marks)**

13. A (1, 4), B (1, 0) and C (3, -2) are three of the vertices of a quadrilateral ABCD. $\overrightarrow{AD} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and X is the midpoint of AC. Find the coordinates of D and X. **(4 marks)**

14. Solve: $6x^2 + x - 2 = 0$. **(3 marks)**

15. (a) Simplify: $36^{½} + 27^{⅔}$. **(2 marks)**

    (b) Solve for x: $4^x = 32$. **(2 marks)**

SECTION B: ATTEMPT THREE QUESTIONS ONLY. **(45 marks)**

16. a) Two lines one passing through points (0, 4) and (3, 1) and the other passing through point (-3, 2), are parallel. Find the equations of these lines. **(7 marks)**

b) If $f(x) = 2x + 3$ and $g(x) = 3x - 1$, calculate:

i) $f(-1)$

ii) $g(-4)$

iii) $fog(x)$

iv) $gof(x)$

v) $gof\left(-\frac{1}{6}\right)$

vi) $fog\left(-\frac{1}{6}\right)$ **(8 marks)**

17. a) A triangle with vertices A, B and C whose coordinates are (2, 0), (5, 4) and (6, 1) respectively is given the translation $t = \binom{-3}{7}$. Find the images of vertices:

i) A', ii) B', iii) C' **(5 marks)**

b) A (-5, -1), B(-2, -1) and C(-4, 2) are vertices of triangle ABC.

i) Plot points A, B and C on a graph paper using a scale of 1 cm to represent 1 unit on each axis. Join the points to form triangle ABC. **(4 marks)**

ii) Triangle ABC is enlarged by a scale factor of -2 with the origin (0, 0) as the centre of enlargement. On the same graph as 17 b(ii) above, draw the image A'B'C' of triangle ABC. **(3 marks)**

iii) Draw triangle A" B" C" which is the image of triangle ABC under a reflection in line y = 0. Use the same graph as that of 17 b (i) above. **(3 marks)**

18. a) Mr. Rwema buys a certain number of pens for £ 1.40 and the number of pence that each one costs him is 4 more than the number of pens that he buys. Find the cost of each pen. £ = pound (British currency) and 1£ = 100 pence. **(5 marks)**

b) Solve: $6x^3 + 11x^2 - 3x - 2 = 0$ **(10 marks)**

19. The table below shows the marks scored by 52 students in a test marked out of 50.

12	18	24	29	37	45	47	38	31	24
19	13	14	20	25	32	39	40	33	25
21	14	40	33	26	21	15	16	22	27
34	41	41	35	27	22	16	17	27	22
28	44	42	35	18	22	28	36	43	18
23	36								

a) Make a grouped frequency table for the marks starting with 12 – 19.

b) Find the modal class and its limits. Calculate the mean. **(15 marks)**

20. a) Given that the points (4, -1), (1, 5) and (-3, k) lie on a straight line calculate the value of K. **(5 marks)**

b) The data below show a relation between x and y.

X	3	4	5	6	7
Y	10	13	16	19	22

By plotting y against x on a graph, determine the gradient of the graph hence deduce the relation between y and x. Write the equation connecting y and x. **(10 marks)**

END

ANSWERS FOR NATIONAL EXAMINATION 2014.

MATHEMATICS I

SECTION A

1. $0.42^2 - 0.58^2 = (0.42 + 0.58)(0.42 - 0.58)$

$= (1.0)(-0.16)$

$= -0.16$

2. $(2\frac{2}{5} \div 1\frac{6}{10}) \times 0.02 = \frac{12}{5} \times \frac{10}{16} \times \frac{2}{100}$

$= \frac{3}{100} = 0.03$

3. Let $g^{-1}(x) = y$; then $g(y) = x$

$3 + 4y = x$

$4y = x - 3$

$y = \frac{x-3}{4}$

4. Let the actual length and width of the piece of land be L and W respectively.

$\frac{L}{15000} \times \frac{W}{15000} = 300\text{cm}^2$

Actual area $= L \times W$

$= 15000 \times 15000 \times 300\text{cm}^2$

$= 67{,}500{,}000{,}000\text{cm}^2$

$= \frac{67{,}500{,}000{,}000}{100{,}000{,}000}$

$= 675\text{ha}$

5. $\frac{2x-5}{x^2-4} = \frac{5}{x-2}$

Restrictions: $x - 2 \neq 0$ and $x + 2 \neq 0$

i.e $x \neq 2$ and $x \neq -2$

$(2x - 5)(x - 2) = 5(x^2 - 4)$

$(2x - 5)(x - 2) - 5(x - 2)(x + 2) = 0$

$(x - 2)(2x - 5 - 5x - 10) = 0$

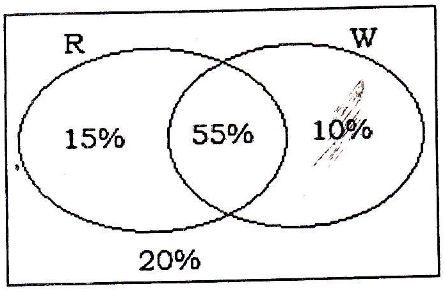
$(x - 2)(-3x - 15) = 0$

$-3x - 15 = 0$

$x = \frac{15}{-3}$

$x = -5$

6. Using a Venn diagram;



Candidates who passed the interview = 55%

Candidates who passed R only = 70% - 55% = 15%

Candidates who passed W only = 65% - 55% = 10%

Candidates who failed the interview

= 100% - (55 + 15 + 10)%

= 20% → 4 candidates

Number of candidates who passed the interview

$= \frac{55 \times 4}{20} = 11$

7. $\begin{cases} 2x + 3y = 5 \ldots (i) \\ 3x + 2y = 10 \ldots\ldots (ii) \end{cases}$

-2x..(i) and 3x..(ii)

$-4x - 6y = -10$

$+\ 9x + 6y = 30$

$5x + 0 = 20$

$5x = 20$

$x = 4$

Substitute x in equation ..(ii); $3x + 2y = 10$

$3 \times 4 + 2y = 10$

$12 + 2y = 10$

$2y = 10 - 12$

$2y = -2$

$y = -1$

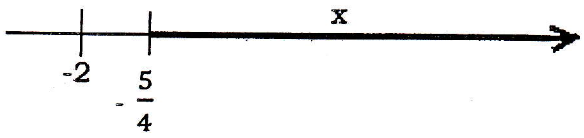
8. $2x - \frac{1}{3}(4x - 1) < \frac{3}{4} + x$

$24x - 16x + 4 < 9 + 12x$

$-4x < 5$

$-x < \frac{5}{4}$

The solution is $\{x: x > -\frac{5}{4}\}$



9. $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\overline{AB} = \sqrt{(3 - (5))^2 + (-2 - 4)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

$= 10$

10. $\vec{z} + 2\vec{w} = \begin{pmatrix}-2\\4\end{pmatrix} + 2\begin{pmatrix}4\\2\end{pmatrix}$

$= \begin{pmatrix}-2\\4\end{pmatrix} + \begin{pmatrix}8\\4\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix}$

$|\vec{z} + 2\vec{w}| = \sqrt{6^2 + 36^2} = \sqrt{100} = 10$

11. a) One ext angle of a pentagon

$= \frac{360°}{5} = 72°$

So one ext angle of a pentagon = 180° - 72

= 108°

The sum if interior angles of a pentagon

= 5 × 108° = 540°

b) $\frac{900°}{90}$ = (right angles)

So $2n - 4 = 10$

$2n = 14$

$n = 7$

The polygon has 7 sides.

12. $x^2 + (x + 3)^2 = 15^2$

$x^2 + x^2 + 6x + 9 - 225 = 0$

$2x^2 + 6x - 216 = 0$

$x^2 + 3x - 108 = 0$

$x(x + 12) -9(x + 12) = 0$

$(x + 12)(x - 9)$

$x - 9 = 0$

$x = 9$

Area of the triangle $= \frac{9cm \times 12cm}{2} = \frac{108cm}{2}$

$= 54cm^2$

13. $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$= \begin{pmatrix}1\\4\end{pmatrix} + \begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}5\\3\end{pmatrix}$

So D is (5, 3)

Hence $x = \left[\frac{1+3}{2}, \frac{4+(-2)}{2}\right]$

= (2, 1)

X = (2, 1)

14. $6x^2 + x - 2 = 0$

$= 6x^2 + 4x - 3x - 2 = 0$

$= 2x(3x + 2) - (3x + 2) = 0$

$= (3x + 2)(2x - 1) = 0$

$= 3x + 2 = 0 \quad 2x - 1 = 0$

$= x = -\frac{2}{3}$ or $x = \frac{1}{2}$

15. a) $36^{0.5} + 27^{2/3} = \sqrt{36 + (\sqrt[3]{27})^2}$

$= 6 + (3)^2 = 15.$

b) $4^x = 32$

$2^{2x} = 2^5$

$2x = 5$

$x = \frac{5}{2} = 2\frac{1}{2}$

SECTION B:

16. a) Gradient $m = \frac{change\ in\ y-values}{change\ in\ x-values}$

$m = \frac{1-4}{3-0} = \frac{-3}{3} = -1$

By using one point above we get the equation of the first line:

$-1 = \frac{y-4}{x-0}$

$-x = y - 4$

$y = -x + 4$

when the two lines are parallel they have they have the same gradient. So the 2nd equation must be on the form $y = -x + c$ by using points (-3, 2) find the values of the y – intercept by substitution.

$2 = -1(-3) + c$

$2 = 3 + c$

$c = 2 - 3 = -1$

therefore $y = -x - 1$.

b)

i) $f(-1) = 2(-1) + 3 = 1$

ii) $g(-4) = 3(-4) - 1 = -13$

iii) $fog(x) = g[f(x)]$

$= g(2x + 3)$

$= 3(2x + 3) - 1$

$= 6x + 9 - 1$

$= 6x + 8$

iv) $gof(x) = f[g(x)]$

$= f(3x - 1) + 3$

$= 6x - 2 + 3$

$= 6x + 1$

v). $gof\left(-\frac{1}{6}\right) = f[g\left(-\frac{1}{6}\right)] = 6\left(-\frac{1}{6}\right) + 1$

$= -1 + 1 = 0$

vi). $fog\left(-\frac{1}{6}\right) = g[f\left(-\frac{1}{6}\right)] = 6\left(-\frac{1}{6}\right) + 8$

$= -1 + 8 = 7$

17. a) Let (x_0, y_0) be an object coordinate which gives a position vector $\begin{pmatrix}x_0\\y_0\end{pmatrix}$ Know that when you translate a triangle, every point undergoes the same change in position vector of each point, to get the image of each point.

i) The image of A(2, 0) will be X_A, y_A

$$\begin{pmatrix}X_A\\y_A\end{pmatrix} = \tilde{t} + \begin{pmatrix}x_0\\y_0\end{pmatrix}$$

$$\begin{pmatrix}X_A\\y_B\end{pmatrix} = \begin{pmatrix}-3\\7\end{pmatrix} + \begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}-1\\7\end{pmatrix}$$

Therefore A' = (-1, 7)

ii) Image of B is B' which coordinates (x_B, y_B)

$$\begin{pmatrix}X_B\\y_B\end{pmatrix} = \begin{pmatrix}-3\\7\end{pmatrix} + \begin{pmatrix}5\\4\end{pmatrix} = \begin{pmatrix}2\\11\end{pmatrix}$$

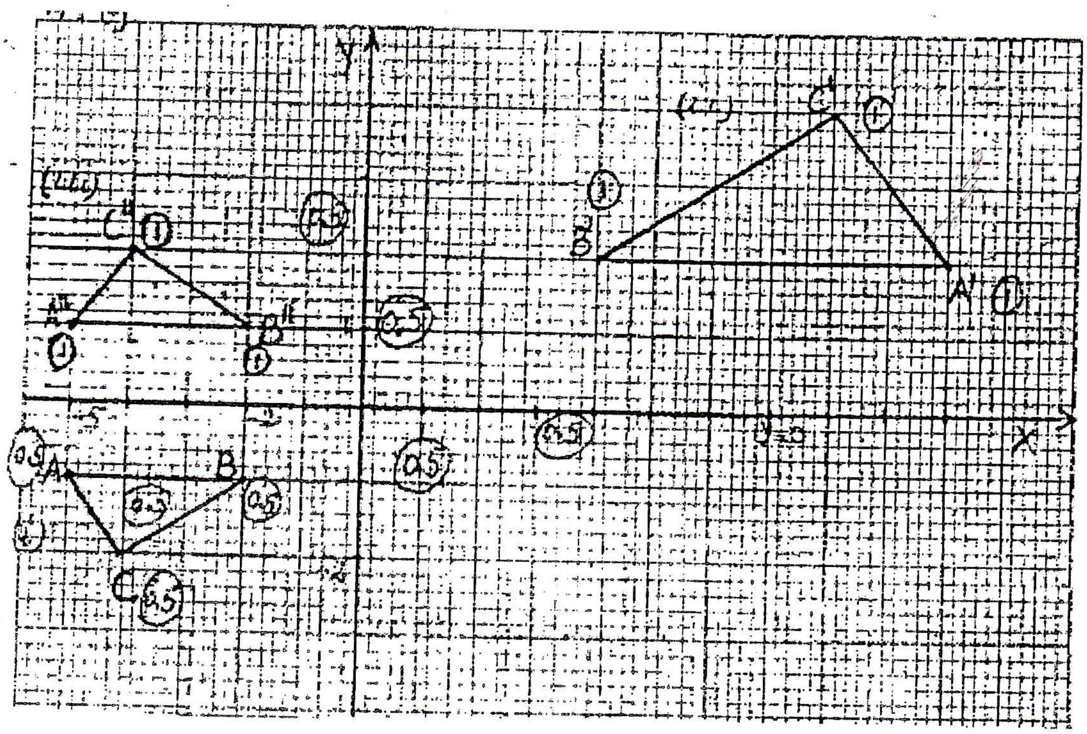
The image of B is at OB' = $\begin{pmatrix}2\\11\end{pmatrix}$ and the coordinates of B' are $(X_B, y_B) = (2, 11)$

iii) Image of C = C' (X_C, y_C)

$$\begin{pmatrix}x_0\\y_0\end{pmatrix} = \begin{pmatrix}-3\\7\end{pmatrix} + \begin{pmatrix}6\\1\end{pmatrix} = \begin{pmatrix}3\\8\end{pmatrix}$$

So $(X_C, y_C) = (3, 8)$

17. b)



18. a) Let the cost of each pen (in p) be x

£1.40 = 140 pence

Number of pens bought = x - 4

$x - 4 = \frac{140}{x}$

$x^2 - 4x - 140 = 0$

$x^2 - 14x + 10x - 140 = 0$

$x(x - 14) + 10(x - 14)$

$(x - 14)(x + 10) = 0$

$x - 14 = 0$

$x = 14$

The cost of each pen is 14 pence.

b) $6x^3 + 11x^2 - 3x - 2 = 0$

$6x^3 + 12x^2$

$-x^2 - 3x$

$-x^2 - 2x$

$-x - 2$

$-x - 2$

0

So $6x^3 + 11x^2 - 3x - 2 = 0$

$\therefore (x + 2) = 0$ or $(2x - 1)(3x + 1) = 0$

$\therefore x + 2 = 0$ or $2x - 1 = 0$ or $3x + 1 = 0$

$\therefore x = -2$ or $x = \frac{1}{2}$ or $x = -\frac{1}{3}$

19.

Class	Mid interval of class, x	Difference from working mean, d	Frequency, f	fd
12 – 19	15.5	- 16	12	-192
20 – 27	23.5	-8	16	-128
28 – 35	31.5	0	10	0
36 – 43	39.5	8	11	88
44 – 51	47.5	16	3	48
			∑f = 52	∑f d= -184

Modal class is 20 - 27

Modal limits = 23.5

Working mean = 13.5

Mean = $31.5 + \left\{-\frac{184}{52}\right\} = 27.96 \approx 28.$

20.

a) $\frac{5-(-1)}{1-4} = \frac{k-(-1)}{-3-4}$

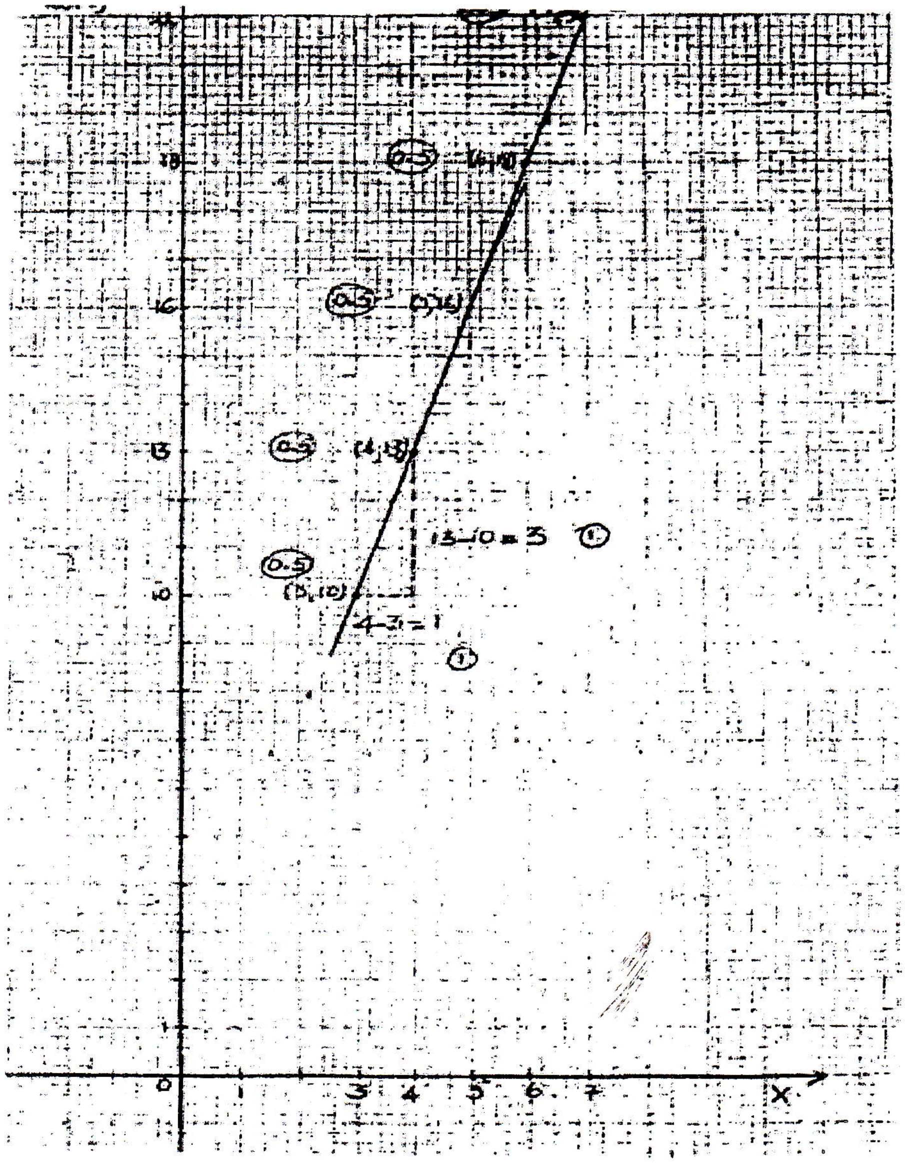
$\frac{6}{-3} = \frac{k+1}{-7}$

$-2 = \frac{k+1}{-7}$

$k + 1 = 14$

$k = 14 - 1 = 13$

b)



Gradient = $\frac{3}{1}$ = 3

Relation between y = $3x$ + c

10 = 3(3) + c

c = 10 - 9 = 1

Equation: y = $3x$ + 1

END